MULTIMODAL EQUILIBRIUM MODELS FOR PREDICTING INTERCITY FREIGHT FLOWS

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ABSTRACT*

In this paper new approaches to the intercity freight transportation system modeling are developed and analyzed. We take an equilibrium supply-demand approach, where the demand side represents the behavior of shippers (cargo owners) and the supply side represents the behavior of carriers (transportation operators). Shippers decisions considered include choice of destination, mode, carrier for pure modes and transfer point for combined modes. Carriers take routing decisions over a multimodal, multiproduct and multioperator network. We develop several simultaneous mathematical formulations to find consistent solutions for flows and level of services, each of them based on particular assumptions on the costs perceptions taken into account by shippers and carriers. Special rationality conditions are shown to be required with respect to fares charged and network routing decisions, in order to be able to obtain consistent system equilibrium solutions. The properties of solutions to the models formulated are derived and analyzed. Finally a solution approach is proposed.

* Project financed by FONDECYT (1950976) and the Catholic University of Chile.

1. INTRODUCTION

Modeling of intercity freight flows has received far less attention than passenger travel demand modeling and, in general, the decision process governing intercity freight markets are less well understood. Many of the studies and models presented in the literature in the past (CACI network model; Bronzini, 1980; Petersen's model, Petersen and Fullerton, 1975; Lansdowne's model; Lansdowne, 1981; Princeton's model, Kornhaunser et al., 1979) adopt a partial approach or introduce important simplifications in their formulations. However, system wide views have also been developed, starting with the Harvard-Brookings model (Kresge and Roberts, 1971), later followed by the Freight Network Equilibrium Model, FNEM (Friesz et al., 1986) and the Multimode Multiproduct Network Assignment Model for Strategic Planning of Freight Flows, STAN (Guelat, Florian and Crainic, 1990). More recently, Hurley and Petersen, 1994 proposed an interesting approach for modeling tariffs charged by transportation carriers, and B. Jourquin and M. Beuthe, 1995, develop a GIS approach to long distance freight transportation network modeling.

The two most important models available today take different modeling approaches. FNEM introduced the explicit consideration of two agents that share the decisions related to the way that the cargo moves on the transportation system: shippers and carriers. Each of these agents is represented by one specific submodel. Both submodels interact aiming to find an equilibrium solution. STAN represents only carrier decisions, starting from trip matrices externally provided as inputs.

Although the importance of carrier decisions is common to the operation of all public transport services, urban transport network models commonly assume that transport services are known and given. Therefore, these models only represent the behavior of individual travelers. FNEM is the first network model that includes explicit modeling of carrier supply decisions. One of the rare examples of equilibrium carrier-users decisions modeling, for urban transport passenger systems can be found in Fernández and Marcotte (1992).

Consistent modeling of shippers and carriers behavior presents special difficulties that took Friesz, Harker and other collaborators to propose a sequential (or two lebel) shipper-carrier model (Friesz et al., 1986, op cit, Friesz and Harker, 1985). Unfortunately, such model formulation, that is solved sequentially, presents two important practical problems: i) it is impossible to guarantee consistent results, given that the iterative solution approach does not, in general, guarantee convergence, ii) given that two network formulations are used, one for shippers and other for carriers, a difficult and generally non-determined transformation must be made to transfer operating costs from the carriers to the shippers network.

Given this, the cited authors proposed simultaneous formulations of the problem. First, Harker and Friesz, (1982), proposed a simultaneous freight network equilibrium model (SFNEM), which considers simultaneous decisions of shippers and carriers while maintaining two different networks. However, the formulation corresponds to a mathematical problem with nonlinear objective function and nonlinear constraints, that requires path enumeration and it is therefore very difficult to solve for real applications. Later, Friesz et al., (1985), proposed a different simultaneous formulation which corresponds to a mathematical problem with nonlinear objective function, but linear constraints. This model keeps the assumption of two different networks, and adds the condition that the transportation (rail[1]) market is perfectly competitive, with prices equal to marginal costs. However, the solution still requires path enumeration, which makes it very difficult to apply to real cases. An additional limitation of all the formulations proposed by these authors is that they only consider carriers with complementary networks. There is no carrier competition over a common infrastructure. The freight rail system in the U.S.A. has carriers operating complementary networks, but that is not the case in other countries[2], where different carriers compete over the same infrastructure.

STAN avoids the carriers-shipper consistency problem by modeling only carrier decisions (routing over a multimodal network). Shipper decisions, associated with demand modeling, are externally provided as trip matrices.

In this paper a different modeling approach, based on a simultaneous demand-supply network equilibrium formulation, is proposed. With that purpose, well-known demand and network models are used within an equilibrium framework, in order to represent shipper and carrier decisions. The model proposed has many important differences with previous models as FNEM and STAN. Only one network is used, over which flows are assigned and transportation level of service is calculated. Supply (carriers) and demand (shippers) models are integrated in a simultaneous mathematical formulation. It is assumed that demand (or shipper) decisions are determined on the basis of level of services corresponding to the transportation alternatives considered. These alternatives are explicitly included in the demand models used. A hierarchical structure is assumed for the demand model. Levels of service are determined by the assignment of flows to the network. The main difference with STAN, is that trip matrices are internally determined by the equilibrium model. Explicit mathematical formulations not known before are developed for different assumptions with respect to congestion. Sufficient conditions for the existence and uniqueness of diagonalized versions of the mathematical problems formulated are given. A general solution approach is proposed.

An additional important result is obtained: In a simultaneous equilibrium formulation of a transportation freight system, two conditions are required in order to obtain a consistent equilibrium solution, i) fares charged by carriers must be consistent with operating costs. If we assume total independence between fares and operating costs, no consistent equilibrium solution can be obtained. However, we show that some rather general rational fare structure allows obtaining consistent equilibrium solutions, ii) carrier network routing decisions, can not be performed without consideration of the effect that they will have on the costs experienced by shippers. In other words, carriers can not have the only objective of minimizing their own operating costs, but must also care about providing a good level of service for their customers. In

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[1] Rail is the only mode explicitly considered

[2] That is the case of truck companies in general. Recently, some rail systems have also started to operate in the same way, with different companies runing trains over a public rail network. That is the case in Chile, since new legislation was pass 5 years ago.

the today world both conditions seem to be reasonable for the sustainable operation of a freight transportation system based on market and commercial principles.

2. MODELING ASSUMPTIONS

We take an equilibrium supply-demand modeling approach in order to simultaneously represent in a consistent way the decisions of shippers and carriers. Shippers decisions are simulated by appropriate distribution and modal split demand models and carriers behavior is simulated by an assignment submodel over a multimodal, multiproduct network with asymmetric costs.

a) Shippers Modeling

We assume that shippers can represent:

- Producers that are distributing their products and therefore sending them to distribution centers (warehouses, or commercial centers), transfer points (ports for exporting), or specific customers.
- Producers acquiring intermediate production inputs.
- Wholesale distributors or retailers purchasing final products.

Shippers decide either where to purchase or where to sell the corresponding products, and in general also how to ship the product (which carrier or combination of carriers to use).[3]

In general, when shippers are purchasing products, they try to buy and transport them in such a way that the commodity price at the origin plus the total (generalized) transportation cost to the destination be minimum. On the other side, if the shipper is distributing products, he will try to maximize the final price obtained at destination less the total transportation cost from the origin. Finally, if the shipper has to send a product between an specific O-D pair he will chose the mode and carriers that minimize the total relevant transportation cost. The behavior of shippers is strongly related to the product transported. An important special case corresponds to shippers that have their own private fleet (mainly trucks), which obviously is a key factor affecting their choice of transport service, as is the case of car ownership in the choice of passengers transport mode.

With respect to the general modeling approach, we assume that shippers decisions of destination, mode and carrier, are better modeled as demand decisions, using distribution and modal split models and that the routing decisions over the network are taken by each individual carrier.

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[3] For relatively small shipments, specially for general cargo, this decision is in general transferred to a professional shipper who decides the best way to send the shipment.

Therefore, both the choice of mode and carrier type are modeled by the modal split submodel. We assume that the use of each mode or carrier reports a utility (equal to a negative generalized cost in this case) to the shipper. This corresponds to the usual approach of random utility theory, where each alternative *i* is represented by a utility U_i that takes the form: $U_i = V_i + \varepsilon_i$, with V_i equal to a function of the observed attributes of alternative *i* and (in our case) the observed characteristics of the product transported and ε_i is the random component derived from unobserved variables that influence the real utility.

Therefore, if a product *p* has to be transported between an O-D pair *w*, where different modes *m* operate, the formulation of V_m should consider variables that define the level of service of transportation alternatives as: the transportation tariff R^p_{wm}, the travel time t^m_w, the percentage of product loses normally observed l^p_{wm} and the dependability of the mode, that can be represented by the standard deviation of travel times distribution σ^t_m. Also, product characteristics as its value for the shipper v^p, and special handling conditions, (refrigerated, fragile, etc.) should be included.

When a combination of modes and carriers is used, we assume that the shipper decides the sequence of modes and the main transfer point where the shipment is transferred between modes, specially if it is a port. We consider that the choice of carrier type is relevant only for road transport (trucks) and train. In general, for a given combination of modes, i.e. train and ship, it is assumed that only a few transfer points (i.e. ports) are relevant and therefore can be identified and enumerated in advance.

Therefore, the demand model includes a transfer choice submodel for the case of combined modes; for an example of a similar approach to modeling of transfer choice in the urban case see: Fernández et al. 1994. This modeling approach is consistent with assuming that in the choice of transfer point between modes and carriers, the shippers consider, in addition to the observable operational characteristics and transfer costs, some non-observable factors that only can be taken into account through an appropriate calibration of a demand choice model.

The shipments (and therefore the shippers) considered are classified according to the following categories:

i) Product transported.
We assume that the characteristics of the product transported strongly influence the transportation decisions taken by the shipper.

ii) Commercial position.
Different criteria will be used to decide decisions related to spatial product distribution if the shipper is a buyer (at destination) or a seller (at the origin) of the product to be distributed (see section 3.2).

iii) Trade type.

We assume that the influence of different factors affecting transportation services consumption will be different if the product is being exported, imported or internally traded.

iv) Shipment size.
Negotiation power and conditions obtained for transporting the products will be in general influenced by the size of the shipments.

v) Fleet ownership.
The fact that a shipper has his own private fleet will also importantly influence the transportation decisions; this requires treating separately to shippers that are in such position.

b) Carriers Modeling

Transportation operators that provide transportation services to the shippers, are called "carriers". They will in general operate a fleet of vehicles corresponding to only one mode; nevertheless, on each mode different carriers can operate, providing different levels of service (different service quality and fare).

Carriers receive requests to transport shipments between O-D pairs, (i,j) that are within the reach of the network that they operate. These origin and destinations could coincide with the primary origin and final destination of the shipment or to intermediate points, where this is transferred between two different carriers.

In general, carriers can offer: i) scheduled services with known routes and frequencies and, ii) services on request by a shipper (for-hire) to satisfy his special transportation needs; this will be performed under an special contract for a given period of time, to transport a predetermined number of product units, between prespecified centroids on the network. iii) in addition we must also consider as a different type the private carrier that transport his own shipments. Carriers are identified by the characteristics of the service offered.

For carriers with scheduled services, freight will be assigned to the corresponding network of services. Alternatively, to carry shipments between a given O-D pair *w*, carriers operating non scheduled services, will choose network routes that minimize their operating costs; therefore in this case vehicles must be assigned to the corresponding modal infrastructure networks.

c) Network and vehicles modeling

The model considers, a set of modes M, with two type of elements: i) pure modes m, with operators $r \in R_m$ and, ii) combined modes $\bar{m}$, made of pure modes joined by a transfer point $d \in D_{\bar{m}}$. Both, combined modes and transfer points to be considered (in addition to pure modes) are predefined as choice alternatives in the demand models. The set of all paths $P_w \in P$ joining a given origin destination pair $w \in W$ is made of: the subsets of paths P_{wmr}, corresponding to each operator r considered for each pure mode m, and the subsets $P_{w\bar{m}d}$, corresponding to the paths going through each transfer point d for each combined mode $\bar{m}$. If any of these subsets contains only one path this could be denominated by p_{wmr} or $p_{w\bar{m}d}$.

The total flow on each arc a of the multimodal network is equal to the sum of flows corresponding to all operators of pure modes, plus the flows corresponding to combined modes using the arc:

$$f_a = \sum_{m \in M} \sum_{r \in R_m} \delta_{amr} \, f_{amr} + \sum_{\bar{m} \in M} \sum_{d \in D_{\bar{m}}} \delta_{a\bar{m}d} \, f_{a\bar{m}d} \tag{1}$$

where

$$\delta_{amr} = \begin{cases} 1 \text{ if arc } a \text{ is used by carrier } r \text{ of pure mode } m \\ 0 \text{ otherwise} \end{cases} \tag{2a}$$

$$\delta_{a\bar{m}d} = \begin{cases} 1 \text{ if arc } a \text{ is used by paths going through transfer point } d \text{ of combined mode } \bar{m} \\ 0 \text{ otherwise} \end{cases} \tag{2b}$$

It is important to notice that transfer points on combined modes are choices equivalent to operators on pure modes.

We assume that one or more types of vehicles v are identified with each of the products p to be transported. Empty vehicles are modeled as a special product (that is carried by all types of vehicles) and therefore, a special demand model is used to generate and distribute empty vehicles. Fleet constraints are considered by definition of a nonlinear increasing waiting time function, in the initial access and transfer arcs, with a capacity in TONs defined by the amount of empty vehicles available, to transport each product (Fernández et al., 1998).

3. DEMAND MODELLING

3.1 Structure of the Demand Decision Process

According with the approach defined in section 2, the shippers decision process with respect to the choice of mode, carrier type, and transfer point in the case of combined modes, is represented through the use of a hierarchical choice model (Williams, 1977). This model has two levels:

i) bottom level: the choice of shipper type, given that a pure mode has been chosen (truck, train or ship), or the choice of transfer point, given that a combined mode has been chosen;

ii) upper level: the choice of mode (pure or combined).

For combined modes, only important transfer points are considered, like ports between train and ship or truck and ship, or main transfer yards between truck and train. Therefore, only combined modes, corresponding to chains containing two stages separated by a transfer will be considered in the modal split model. If the chain contains more than two modes and therefore more than one transfer, only the choice of the most important transfer will be considered in the demand model; it is assumed that the choice of the other less important transfers will be made by the carriers in charge of delivering the shipment and therefore will be modeled as routing decisions over the appropriate combined network. Therefore the networks definition, for the assignment step should be defined consistently with these considerations. The general structure of model corresponds to a desaggregate hierarchical logit model (Williams, 1977) and is shown in Figure 1.

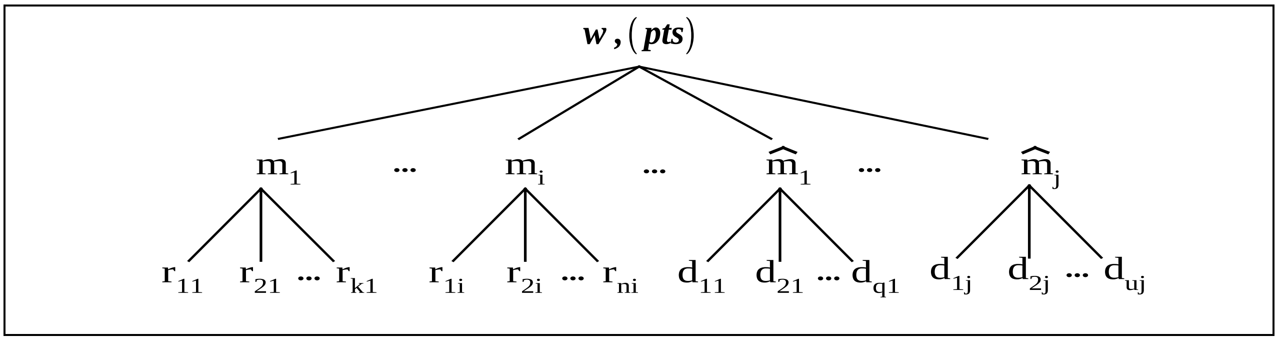

Figure 1. General structure of the mode, carrier and transfer point choice model.

3.2 Trip Generation

The model proposed corresponds to a short run equilibrium model, therefore we assume that both prices of commodities and total supply and demands at different points in the territory are given. Then, the total origins O_i^{pts} and destinations D_j^{pts} corresponding to each product *p*, trade type *t* and shipment size *s*, at each relevant centroid in the network are known inputs to the model.

Nevertheless some special care must be taken to consistently specify this data. Many important products normally considered, in freight transport models are interrelated because they belong to a same production chain. Thus, we can start with some row materials that are extracted at a certain location and transported to other location, where they are inputted to a production process that transform them in a different product; this in turn can be transported to a new production unit, located in a different place and inputted in a new production process and thus several times until obtaining a final product, that is transported to distribution centers, for their final commercialization through wholesalers or retailers. Therefore, the O_i^p and D_j^p for different products, belonging to the same production chain, must satisfy input-output constraints given by the corresponding production process.

In developed countries these production chains can be long and complex making the analysis more difficult than in developing countries where the production chains are generally short and simple. (i.e. production chains of the sugar, the cement, the iron, the pulp etc.); in the case of row materials that are exported this chains can be formed by only one or at most two stages.

Some care must also be taken for the case of seasonal products, depending on the modeling period defined. Thus, a different input-output coefficient should be considered for the relation between the input to the production of sugar (beet for example) if the modeling period is a year, or corresponds only to the months in which such agricultural input is harvested. In this last case it must be taken into account that during the period of few months considered, the amount of beet transported will correspond to the total required for the yearly production of sugar, however the amount of sugar produced and transported out of the plant will be proportional only to the months considered.

3.3 Trip Distribution Modeling

Simple and double constrained aggregate gravity expressions are used for distribution, and different models must be calibrated depending on the specific product *p*, trade type *t*, and shipment size *s*. Also, different formulations are used for the cases of shippers buying products at destinations (or importers) and shippers selling products at origins (or exporters). For double constrained gravity models the following general expression is used:

$$T_w^{pts} = A_i^{pts} O_i^{pts} B_j^{pts} D_j^{pts} \exp\left(\beta^{pts} L_w^{pts} + q^{pts} P_i^p\right) \tag{3}$$

where, β^{pts} is the distribution model parameter for product *p*, trade type *t* and shipment size *s*, P_i^p is the price of product *p* at origin *i*; by construction of the hierarchical choice model described in Figure 1, β^{pts} corresponds also to the parameter "phi"associated to the highest choice level (trip distribution). *q* is a conversion parameter expressed in utility units per dollar[3]. L_w^{pts} is the "expected maximum utility" (EMU), for shipments between O-D pair (*i*,*j*). The formulation of the EMU is given by the so called "logsum" of the corresponding modal utilities $\tilde{V}_{wk}^{pts}$ introduced in section 2-a (Williams, 1977):

$$L_w^{pts} = \ln \sum_{k \in M} \exp\left(\beta_k^{pts} \tilde{V}_{wk}^{pts} + \alpha_k^{pts}\right) \tag{4}$$

where β_k^{pts} is the parameter of the EMU variable which represents the composite utility of the nest under mode *k*, in the modal choice level of the choice hierarchy (see, Fig. 1). A_i^{pts} and B_j^{pts} are the balancing factors whose expressions are:

$$A_i^{pts} = \frac{1}{\sum_j B_j^{pts} D_j^{pts} \exp\left(\beta^{pts} L_w^{pts} + q^{pts} P_i^p\right)} \tag{5}$$

$$B_j^{pts} = \frac{1}{\sum_i A_i^{pts} O_i^{pts} \exp\left(\beta^{pts} L_w^{pts} + q^{pts} P_i^p\right)} \tag{6}$$

The gravity model (3) represents the origin destination distribution decisions made by shippers. It assumes that the influence of generalized transportation costs $\tilde{V}_{wk}^{pts}$ (or L_w^{pts}) on these decisions depends on the product considered, the trade type and the shipment size; therefore, different models (or different parameters β^{pts}) must be calibrated for each of these cases.

In expressions (3), (5) and (6) of the distribution model, it is implicitly assumed that the shipper is interested in minimizing the delivered price of product *p* at destination *j*, given that the distribution decisions depend on the final price $\beta^{pts} L_w^{pts} + q^{pts} P_i^p$ (see section 2-a). Alternatively for shippers selling products at origins (or exporters) the objective will be to maximize the final price of the product at destination *j* less the equivalent transportation cost from the origin of the product *i* to the destination *j*. In such case the exponent of the distribution model should be $\left(\beta^{pts} L_w^{pts} - q^{pts} P_j^p\right)$.

3.4 Modal, Carrier and Transfer Choices

For the case of pure modes *m* (trucks and train), the following model is used in order to determine the proportion of shipments type (*pts*), traveling between O-D pair *w*, by carrier type *r*:

$$G_{wmr}^{pts}\left(\vec{V}_{wm}^{pts}\right) = \frac{\exp\left(\gamma_m^{pts} V_{wmr}^{pts}\right)}{\sum_{k \in R_m} \exp\left(\gamma_m^{pts} V_{wmk}^{pts}\right)} \tag{7}$$

where V_{wmr}^{pts} is the observed (or systematic) utility (or negative generalized cost) of using carrier type *r* of mode *m* between O-D pair *w*, for product type *p*, trade type *t* and shipment size *s*; $\vec{V}_{wm}^{pts}$ is the vector of observed utilities corresponding to all carriers available for traveling in mode *m* between O-D pair *w*.

For the case of combined modes $\bar{m}$ the following model is used to determine the proportion of shipments type (*pts*) going through transfer point *d* to travel between O-D pair *w*:

$$G_{w\bar{m}d}^{pts}\left(\vec{V}_{w\bar{m}}^{pts}\right) = \frac{\exp\left(\gamma_{\bar{m}}^{pts} V_{w\bar{m}d}^{pts}\right)}{\sum_{k \in D_{\bar{m}}} \exp\left(\gamma_{\bar{m}}^{pts} V_{w\bar{m}k}^{pts}\right)} \tag{8}$$

In this case $V_{w\bar{m}d}^{pts}$, represents the shipper's perception of the generalized travel cost for combined mode $\bar{m}$ via transfer node *d*. This is made up of the costs incurred over each modal section plus the transfer costs.

A general expression of the generalized costs corresponding to specific carriers or transfer points is:

$$V_{wmr}^{pts} = -\left(\alpha_{mr}^{pts} + R_{wmr}^{pts} + \theta^{pts} t_{wmr} + \gamma_{l}^{pts} l_{wmr} + \gamma_{\sigma}^{pts}\, \sigma_{wmr}\right) \tag{9a}$$

$$V_{w\vec{m}d}^{pts} = -\left(\alpha_{\vec{m}d}^{pts} + R_{w\vec{m}d}^{pts} + \theta^{pts} t_{w\vec{m}d} + \gamma_{l}^{pts} l_{w\vec{m}d} + \gamma_{\sigma}^{pts}\, \sigma_{w\vec{m}d}\right) \tag{9b}$$

where α, θ and the different γ are parameters to be calibrated on observed data in order to adjust the model to observed behavior with respect to the choice process for carrier and transfer point: these parameters represent the relative attractiveness of specific carriers or transfer points, due to factors not represented by the level of service variables explicitly included in the generalized cost expression. These variables are: i) the transport fare *R*, for the corresponding pair *w* and carrier *r*, or transfer *d*; notice that, in the case of transfer choice (9b), it includes the fares corresponding to each mode segment of the combined mode (from the origin to the transfer point *d* and from the transfer to the destination), plus the transfer fare, if this exists, ii) the travel time *t*, for the corresponding O-D pair *w* and carrier *r*, in the case of pure mode *m*, or for transfer point *d* in the case of combined mode $\vec{m}$, considering the travel times on each segment of the trip. iii) the percentage of product loses *l*, experienced in option, and iv) the variability of travel times σ, on each option, which can be calculated as the variance of the values of travel times observed.

The proportion of shipments (*pts*) using each mode *m* (pure or combined) when traveling between O-D pair *w* is given by:

$$G_{wm}^{pts}\left(\vec{\tilde{V}}_{w}^{pts}\right) = \frac{\exp\left(\beta_{m}^{pts}\tilde{V}_{wm}^{pts} + \alpha_{m}^{pts}\right)}{\sum_{k\in M}\exp\left(\beta_{k}^{pts}\tilde{V}_{wk}^{pts} + \alpha_{k}^{pts}\right)} \tag{10}$$

where, for pure modes, $\tilde{V}_{wm}^{pts}$ is the EMU, of the utilities corresponding to all relevant carriers of mode *m*, available between O-D pair *w* to transport shipments (*pts*), $r \in R_{m}^{pts}$, and for combined modes, is the "EMU" of the utilities corresponding to all relevant transfer points available between O-D pair *w*, to carry shipments (*pts*) by mode $\vec{m}$, $d \in D_{\vec{m}}^{pts}$.

$$\tilde{V}_{wm}^{pts} = \frac{1}{\gamma_{m}^{pts}} \ln \sum_{r\in R_m} \exp\left(\gamma_{m}^{pts} V_{wmr}^{pts}\right) \tag{11a}$$

$$\tilde{V}_{w\vec{m}}^{pts} = \frac{1}{\gamma_{\vec{m}}^{pts}} \ln \sum_{d\in D_{\vec{m}}} \exp\left(\gamma_{\vec{m}}^{pts} V_{w\vec{m}d}^{pts}\right) \tag{11b}$$

The above models (9), (10) and (11), should be calibrated using observations from individual shippers choices of mode, transfer point and carrier type when appropriate. In principle, as many different models as the number of products defined, times the number of trade types and times the number of shipments sizes considered, should be calibrated. However in order to make more efficient the use of information, the calibration of different models could be pooled together, by using dummy variables to distinguish among products and shipment sizes, treating the rest of variables as generic. The appropriate approach and the final number of models calibrated depends on practical considerations with respect to the data available and the experimental results obtained with alternative formulations.

The demand model considering both distribution and modal split (included, mode, carrier and transfer point) will then have the general form:

$$T_{wmr}^{pts} = T_{w}^{pts} \cdot G_{wm}^{pts} \cdot G_{wmr}^{pts} \qquad (12a)$$

$$T_{wm\square d}^{pts} = T_{w}^{pts} \cdot G_{wm\square}^{pts} \cdot G_{wm\square d}^{pts} \qquad (12b)$$

4. SUPPLY MODELING

4.1 General Considerations

Carriers take the network operations, including vehicle scheduling and detailed routing decisions. According with the formulation of the modal split model presented in section 3.4, the only routing related decision, on the multimodal multicarrier network, reserved to the shippers, is the selection of transfer point for combined modes, and the possibility of using scheduled services, which are identified as a different carrier option in the lower level of the modal split model for "pure" modes.

To be consistent with the behavioral principles assumed and the formulation of the demand models described before, a different network should be defined for each pure mode and for the combinations with transfers not resolved by the demand model. However, the transportation costs on the arcs should be function of the flows corresponding to all the carriers that share the use of the same arc. Therefore, because different carriers will use different type of vehicles, with different congestion impacts, we will have a network assignment problem with asymmetric costs.

All the vehicles operating on scheduled services should be preassigned to the corresponding infrastructure networks and the cost functions accordingly redefined (see, Fernández and the Cea, 1990). Then, for these services an appropriate service network based on the concept of route sections (Fernández and the Cea, 1993) should be constructed in order to assign the corresponding matrix of shipments (physical units of the corresponding products), that use this type of services, obtained from the modal split model.

It is necessary that the O-D matrices by product, obtained from the demand models be aggregated over shipment size, trade type and identified with the corresponding vehicle type *v* for non scheduled services, before the assignment. Thus, consolidated vehicle matrices, by vehicle type, will be assigned to the multimodal network, with the exception of matrices corresponding to carriers with scheduled services[4], that must be kept in tons, and assigned to the corresponding service netwoks.

4.2 Network Equilibrium Conditions

The equilibrium principle used to describe the use of the network, and therefore obtaining the freight flows assignment, depends on the type of carrier considered:

i) For carriers operating scheduled services the corresponding O-D matrices are assigned to the network of services assuming that the routing decisions are consistent with Wardrop's system optimization principle; therefore, at equilibrium marginal costs will have the same value on all used routes, and they will be lower than those that would be experienced on the alternative non-used routes.

ii) For carriers operating non-scheduled services that must share the use of infrastructure with other carriers (roads, rails, ports, etc.), the corresponding vehicles matrices are assigned to the network using a Wardrop type equilibrium principle, but based on the consideration of the private marginal cost experienced by the carrier. Notice that in this case private and social marginal costs are different, because the first considers the congestion effects only over the own vehicles operating costs and the second takes into account the effect over all the vehicles operating on the same infrastructure (see, Harker, 1988). An special case of interest corresponds to a carrier operating only one vehicle, because then the private marginal cost is equal to the private average cost.

Alternative model formulations are defined considering different components on the cost function taken into account by the carriers for routing decisions.

The following, are considered as possible components of the cost function used by the carriers for routing decisions: c_p is the average vehicle operating cost over route p, that is assumed to be independent of flow, t_p, is the average travel time over route p, which can depend on flows magnitude, $\overline{tM}_p$, is the private marginal travel time; we also define as $\overline{CM}_p = c_p + \phi\, \overline{tM}_p$, the total marginal private cost perceived by the operator, and by $C_p = c_p + \phi\, t_p$, the average (private) total operating cost where ϕ represents the value of time for the carrier which in general will be related to the vehicle capital costs involved.

For combined mode networks that consider modes operating according to different principles (private vs. system optimum) the operating costs are calculated over the combined network, but using private costs (average or private marginal) for the arcs on the network operating according to user optimum, and marginal costs for the arcs on the network operating according to system optimum. In all cases the costs are calculated considering all the flows by all types of vehicles

[4] However, different modes have different treatments, for instance for rail, the Tons. of products must be transformed to railcars before assignment.

sharing the same infrastructure. As a way of simplifying notation, for the case of combined modes, the values of $c_p, t_p, \overline{tM}_p, C_p, \overline{CM}_p$, include the costs and/or travel times over the route segments corresponding to both component modes and the transfer point.

4.3 Fare Structure

In this work, we assume that the fare agreed between carrier and shipper, is in general equal to a value by unit of freight transported and is different for each product p and O-D pair $w \in W$. Following Hurley and Petersen (1994a and 1994b) the fare values are composed of two parts: cost and benefit. The cost corresponds to the marginal cost perceived by the carrier and the benefit is equal to a constant value ε, obtained by unit transported. According with Hurley and Petersen, this type of fare structure represents most of the real cases, when two agents interact vertically, as is the case of shippers and carriers.

Additionally, the type of fare assumed, avoids inconsistencies between the costs perceived by shippers and carriers, and used as the basis for their decisions, which is of fundamental importance in a simultaneous equilibrium approach as taken in this work. Therefor we will assume that the fares charged by carriers can in general be represented as:

$$R_{wmr}^{pts} = C_{wmr}^{pts} + \varepsilon_{wmr}^{pts} \tag{13a}$$

$$R_{wm\rceil d}^{pts} = C_{wm\rceil d}^{pts} + \varepsilon_{wm\rceil d}^{pts} \tag{13b}$$

5. ATERNATIVE MODEL FORMULATIONS

In a simultaneous equilibrium formulation, like the proposed in this paper, the complexity of the model and the possibility of obtaining consistent unique solutions and developing convergent algorithms are difficulted by the existence of interactions among the different submodels included in the formulation. For the class of models considered, submodels interactions appear as a consequence of the existence of congestion effects. This is because then levels of service (that influence demand decisions) depend on network flows, which are obtained as a result of routing decisions (simulated by the assignment submodel).

Also, for freight transportation systems, vehicle-operating costs must be in general considered, in addition to travel times, and fares (factors influencing shipper decisions). Operating costs are the most important factor for freight transportation operator (carrier) decisions. However, given that we are using a simultaneous equilibrium formulation, it is required that fares charged by carriers be consistent with operating costs. As we will see in the following sections, if we assume total independence between fares and operating costs no consistent equilibrium solution can be obtained. This is an important result for freight transportation modeling. In section 7.4 it is shown that the fare structure proposed by Hurley and Petersen (1994a) and represented by (13a,b) allows obtaining consistent equilibrium solutions.

Given the preceding discussion, it is possible to make different assumptions with respect to the extent that congestion is relevant for the operation of the system. In the following, three alternative basic formulations are developed, based on different assumptions with respect to the consideration of congestion:

i) Model 1. Congestion is not considered relevant and therefore travel times are taken as independent of flows, both in the simulation of shippers and carriers behavior.

ii) Model 2. Only a partial consideration of congestion is made. Operational carrier decisions are based only on operating costs, which (as we saw before) are independent of flows. Vehicle travel times depend on flows, but they are not taken into consideration for carriers routing decisions. However, shippers perceive the effect of carriers routing decisions on travel times and take them into account for their choices; this means that demand models must consider travel times calculated considering the congestion effects produced by the flows obtained as a consequence of carriers routing decisions (and also carriers choices).

iii) Model 3. Full consideration of congestion is made. Both carriers and shippers take into account travel times for their decisions and their values depend on flow magnitudes. Shippers have a value of time θ^{pts} that depends on the product *p*, trade type *t*, and shipment size *s*.

For each of these three basic models, a different mathematical formulation is developed in the following sections. It can be said that the practical applicability of each model will depend on the relevance of congestion in the system analyzed. It is also important to notice that models 1 and 2 are especial cases of model 3. Also, model 1 is an especial case of model 2.

5.1 Model 1: Without Congestion

In a system without congestion travel times depend only on the route chosen, but are independent of flows; therefore, they are constant for each given route in the networks considered. The most general formulation of the network equilibrium conditions corresponding to this model is the following:

$$c_p^* + \phi\, t_p^* + \theta^{pts} t_p^* \begin{cases} = {U_{wmr}^{pts}}^* \text{ if } {h_p^{pts}}^* \geq 0 \quad \text{i.e. if } {h_p^{pts}}^* = {T_{wmr}^{pts}}^* \\ > {U_{wmr}^{pts}}^* \text{ if } {h_p^{pts}}^* = 0 \end{cases} \qquad \begin{matrix} w \in W \\ r \in R_m \\ p \in P_{wmr} \end{matrix} \tag{14}$$

$$c_p^* + \phi\, t_p^* + \theta^{pts} t_p^* \begin{cases} = {U_{w\bar{m}d}^{pts}}^* \text{ if } {h_p^{pts}}^* \geq 0 \quad \text{i.e. if } {h_p^{pts}}^* = {T_{w\bar{m}d}^{pts}}^* \\ > {U_{w\bar{m}d}^{pts}}^* \text{ if } {h_p^{pts}}^* = 0 \end{cases} \qquad \begin{matrix} w \in W \\ d \in D_{\bar{m}} \\ p \in P_{w\bar{m}d} \end{matrix} \tag{15}$$

Conditions (14) and (15) indicate that the unique route used between each O-D pair *w*, by each carrier *r* of each mode *m*, is that presenting the minimum generalized cost, obtained as the sum of the carrier operating and travel time costs plus the shippers travel time cost perception. Special cases of this formulation can be obtained assigning a zero value to any of the value of time parameters, ϕ or θ, or to both of them.

Given that in this model both shippers and carriers do not consider the congestion effects of network flows on travel times, there is no interaction between demand (shippers) and supply (carriers) submodels.

5.2 Model 2: Partial Consideration of Congestion

The influence of flows on travel time values is only considered by shippers. Carriers routing decisions are based only on vehicle operating costs. Network equilibrium conditions corresponding to this model are the following:

$$c_p^* \begin{cases} = U_{wmr}^* \text{ if } p = p_{wmr}^* \\ \geq U_{wmr}^* \text{ if } p \neq p_{wmr}^* \end{cases} \qquad \begin{matrix} w \in W \\ r \in R_m \\ p \in P_{wmr} \end{matrix} \tag{16}$$

$$c_p^* \begin{cases} = U_{w\bar{m}d}^* \text{ if } p = p_{w\bar{m}d}^* \\ \geq U_{w\bar{m}d}^* \text{ if } p \neq p_{w\bar{m}d}^* \end{cases} \qquad \begin{matrix} w \in W \\ d \in D_{\bar{m}} \\ p \in P_{w\bar{m}d} \end{matrix} \tag{17}$$

where p^* is the unique route chose between each O-D pair w.

Notice that in this model (as in model 1) shippers choices, simulated using the demand models, do not affect carriers routing decisions, given that they are independent of flows; in other words carriers routing decisions are invariant to changes in shippers transportation service choices. Therefore this is equivalent to having an exogenous network routing model, where the routes chosen over the network are not affected by the solutions (trip matrices) obtained from the demand models. However, the magnitude of flows assigned to each route, on each modal network will depend on shipper choices. Then, although shippers decisions will not affect carriers routing decisions, given that congestion effects are considered to determine the level of service (travel times) of the different alternatives considered by shippers, their decisions will affect travel times, producing a feedback effect on the choices simulated by the demand models. This produces an interaction between demand (shippers) and supply (carriers) submodels, that do not exist in the case of model 1, and therefore requires a more sophisticated model formulation and solution approach.

5.3 Model 3: Full Consideration of Congestion

In this model, congestion effects on travel times are considered both by shippers and carriers. As in model 1, it is in addition assumed that carriers take into account valuations of travel times, for their routing network decisions; in other words, they are sensitive to the shippers demand characteristics in order to improve the chances that their transportation services be used by them. Network equilibrium conditions corresponding to this model are the following:

$$c_p^* + \phi\, \overline{tM}_p^* + \theta^{\,pts} t_p^* \begin{cases} = U_{wmr}^{pts\,*} \text{ if } h_p^{pts\,*} > 0 \\ \geq U_{wmr}^{pts\,*} \text{ if } h_p^{pts\,*} = 0 \end{cases} \quad \begin{matrix} w \in W \\ r \in R_m \\ p \in P_{wmr} \end{matrix} \tag{18}$$

$$c_p^* + \phi\, \overline{tM}_p^* + \theta^{\,pts} t_p^* \begin{cases} = U_{w\vec{m}d}^{pts\,*} \text{ if } h_p^{pts\,*} > 0 \\ \geq U_{w\vec{m}d}^{pts\,*} \text{ if } h_p^{pts\,*} = 0 \end{cases} \quad \begin{matrix} w \in W \\ d \in D_{\vec{m}} \\ p \in P_{w\vec{m}d} \end{matrix} \tag{19}$$

Notice that carriers perceive private marginal times $\overline{tM}_p$ but shippers perceive average private times t_p.

The consideration of shippers travel time costs in carriers decisions is required to obtain consistent equilibrium solutions, in the context of a simultaneous supply-demand equilibrium model. This is shown below in section 7.4, when we derive the necessary conditions corresponding to the solution of the equivalent variational problem, that produce the model equilibrium conditions. This condition implies that carriers can not have the only objective of minimizing their own operating costs, but must also care about providing a good level of service for their customers. Therefore, if shippers take into account his own valuation of travel time cost, to decide the choice of carrier, it is rational that the later also considers it to take their operational decisions. Consistently, the condition can only be eliminated if travel time cost is also eliminated from the definition of the shippers generalized travel cost, V_{wmr}^{pts}, $V_{w\vec{m}d}^{pts}$ (see, 9a, 9b).

6. MATHEMATICAL FORMULATIONS

In order to simplify notation, the formulations will consider only one product *p*, one shipment size *s* and one type of commercial interchange *t*. In addition, two generic modes, one pure mode *m* and one combined mode $\vec{m}$, are considered; combined mode $\vec{m}$ is composed of pure modes *m* and $\overline{m}$ which are joined by several transfer points $d \in D_{\vec{m}}$.

Model 1 presents travel times and operating costs independent of flows and therefore constant for each given network route; in this case there is an equivalent optimization problem that satisfies the equilibrium conditions presented in sections 3.3, 3.4 and 5.1. However, models 2 and 3 do not have an equivalent optimization problem, given the asymmetric relations among flow dependent delay functions, for different carriers. In those cases the corresponding equilibrium are satisfied by the diagonalized problems corresponding to each formulation.

6.1 Model 1: Without Congestion

Equilibrium conditions (3), (7), (8), (10), (14) and (15) are obtained solving the following equivalent optimization problem:

(P1):

$$
\begin{aligned}
\min Z = & \sum_{a\in A_m}\sum_{r\in R_m} c_{amr}\, f_{amr} + \sum_{a\in\left(A_m\cup A_{\bar{m}}\right)}\sum_{d\in D_{\bar{m}}} c_{a\bar{m}d}\, f_{a\bar{m}d} \\
& + \phi\sum_{a\in A_m}\sum_{r\in R_m} t_{amr}\, f_{amr} + \phi\sum_{a\in\left(A_m\cup A_{\bar{m}}\right)}\sum_{d\in D_{\bar{m}}} t_{a\bar{m}d}\, f_{a\bar{m}d} \\
& + \theta\sum_{a\in A_m}\sum_{r\in R_m} t_{amr}\, f_{amr} + \theta\sum_{a\in\left(A_m\cup A_{\bar{m}}\right)}\sum_{d\in D_{\bar{m}}} t_{a\bar{m}d}\, f_{a\bar{m}d} \\
& + \frac{1}{\beta_m}\sum_w T_{wm}\left(\ln T_{wm} - 1 - \alpha_m\right) + \frac{1}{\beta_{\bar{m}}}\sum_w T_{w\bar{m}}\left(\ln T_{w\bar{m}} - 1 - \alpha_{\bar{m}}\right) \\
& - \frac{1}{\gamma_m}\sum_w T_{wm}\left(\ln T_{wm} - 1\right) + \frac{1}{\gamma_m}\sum_w\sum_r T_{wmr}\left(\ln T_{wmr} - 1\right) \\
& - \frac{1}{\gamma_{\bar{m}}}\sum_w T_{w\bar{m}}\left(\ln T_{w\bar{m}} - 1\right) + \frac{1}{\gamma_{\bar{m}}}\sum_w\sum_d T_{w\bar{m}d}\left(\ln T_{w\bar{m}d} - 1\right) \\
& + \sum_w\sum_r T_{wmr}\left(\alpha_{mr} + \varepsilon_{wmr} + \gamma_l\, l_{wmr} + \gamma_\sigma\, \sigma_{wmr}\right) \\
& + \sum_w\sum_d T_{w\bar{m}d}\left(\alpha_{\bar{m}d} + \varepsilon_{w\bar{m}d} + \gamma_l\, l_{w\bar{m}d} + \gamma_\sigma\, \sigma_{w\bar{m}d}\right) \\
& + \frac{1}{\beta}\sum_w T_w\left(\ln T_w - 1 - qP_i\right) - \sum_w T_w\left(\ln T_w - 1\right)
\end{aligned} \tag{20}
$$

s.t.

$$T_w = T_{wm} + T_{w\bar{m}} \quad \forall w\in W \tag{21}$$

$$T_{wm} = \sum_r T_{wmr} \quad \forall w\in W \tag{22}$$

$$T_{w\bar{m}} = \sum_d T_{w\bar{m}d} \quad \forall w\in W \tag{23}$$

$$T_{wmr} = \sum_{p\in P_{wmr}} h_p \quad \forall w\in W, \forall r\in R_m \qquad \left(U_{wmr}\right) \tag{24}$$

$$T_{w\bar{m}d} = \sum_{p\in P_{w\bar{m}d}} h_p \quad \forall w\in W, \forall d\in D_{\bar{m}} \qquad \left(U_{w\bar{m}d}\right) \tag{25}$$

$$O_i = \sum_j T_w \quad \forall i\in N_m \qquad \left(\lambda_i\right) \tag{26}$$

$$D_j = \sum_i T_w \quad \forall j\in\left\{N_m\cup N_{\bar{m}}\right\} \qquad \left(\lambda_j\right) \tag{27}$$

$$f_{amr} = \sum_w\sum_{p\in P_{wmr}} \delta_{ap} h_p \quad \forall a\in A_m, \forall r\in R_m \tag{28}$$

$$f_{a\bar{m}d} = \sum_{w} \sum_{p \in P_{w\bar{m}d}} \delta_{ap} h_p \quad \forall a \in \left\{ A_m \cup A_{\bar{m}} \right\}, \forall d \in D_{\bar{m}} \tag{29}$$

$$\delta_{ap} = \begin{cases} 1 \text{ if } a \in p \\ 0 \text{ if } a \notin p \end{cases} \quad \forall a \in \left\{ A_m \cup A_{\bar{m}} \right\}, \forall p \in P_w, \forall w \in W \tag{30}$$

$$h_p \geq 0, T_w \geq 0, T_{wm} \geq 0, T_{w\bar{m}} \geq 0, T_{wmr} \geq 0, T_{w\bar{m}d} \geq 0 \quad \forall p \in P_w, \forall w \in W, \forall r \in R_m, \forall d \in D_{\bar{m}} \tag{31}$$

6.2 Model 2: Partial Consideration of Congestion

Equilibrium conditions (3), (7), (8), (10), (16) and (17) are obtained from the diagonalized version, by operator, of the following formulation:

(P2):

$$\sum_{a \in \left(A_m \cup A_{\bar{m}} \right)} \vec{C}_a \left(\vec{F}_a^* \right)^T \cdot \left(\vec{F}_a - \vec{F}_a^* \right) - \sum_{w \in W} \vec{g}_w \left(\vec{T}_w^* \right)^T \cdot \left(\vec{T}_w - \vec{T}_w^* \right) \geq 0 \quad \forall \vec{F}, \vec{T} \in \Omega \tag{32}$$

where:

$$\vec{C}_a \left(\vec{F}_a \right)^T = \left[\ldots, \theta\, t_{akl} \left(\vec{F}_a \right), \ldots \right]^T \tag{33}$$

$$\vec{g}_w \left(\vec{T}_w \right)^T = \left[-\frac{1}{\beta} \left(\ln T_w - qP_i \right) + \ln T_w ; \ldots, -\frac{1}{\beta_k} \left(\ln T_{wk} - \alpha_k \right) + \frac{1}{\gamma_k} \ln T_{wk}, \ldots ; \right.$$

$$\left. \ldots, -\frac{1}{\gamma_k} \ln T_{wkl} - \alpha_{kl} - R_{wkl} - \gamma_l\, l_{wkl} - \gamma_\sigma\, \sigma_{wkl}, \ldots \right]^T \tag{34}$$

Ω :

$$T_w = T_{wm} + T_{w\bar{m}} \quad \forall w \in W \tag{35}$$

$$T_{wm} = \sum_{r} T_{wmr} \quad \forall w \in W \tag{36}$$

$$T_{w\bar{m}} = \sum_{d} T_{w\bar{m}d} \quad \forall w \in W \tag{37}$$

$$O_i = \sum_{j} T_w \quad \forall i \in N_m \qquad \left(\lambda_i \right) \tag{38}$$

$$D_j = \sum_{i} T_w \quad \forall j \in \left\{ N_m \cup N_{\bar{m}} \right\} \qquad \left(\lambda_j \right) \tag{39}$$

$$f_{amr} = \sum_{w} \delta_{ap_{wmr}} T_{wmr} \quad \forall a \in A_m, \forall r \in R_m \tag{40}$$

$$f_{amr} = \sum_{w} \delta_{ap_{w\bar{m}d}} T_{w\bar{m}d} \quad \forall a \in \left\{ A_m \cup A_{\bar{m}} \right\}, \forall d \in D_{\bar{m}} \tag{41}$$

$$\delta_{ap_{wmr}} = \begin{cases} 1 \text{ if } a \in p_{wmr} \\ 0 \text{ if } a \notin p_{wmr} \end{cases} \quad \forall a \in A_m, \forall w \in W \tag{42}$$

$$\delta_{ap_{w\bar{m}d}} = \begin{cases} 1 \text{ if } a \in p_{w\bar{m}d} \\ 0 \text{ if } a \notin p_{w\bar{m}d} \end{cases} \quad \forall a \in \left\{A_m \cup A_{\bar{m}}\right\}, \forall w \in W \tag{43}$$

$$T_w \geq 0, T_{wm} \geq 0, T_{w\bar{m}} \geq 0, T_{wmr} \geq 0, T_{w\bar{m}d} \geq 0 \quad \forall w \in W, \forall r \in R_m, \forall d \in D_{\bar{m}} \tag{44}$$

6.3 Model 3: Full Consideration of Congestion

Equilibrium conditions (3), (7), (8), (10), (18) and (19) are obtained from the diagonalized version, by operator, of the following formulation:

(P3):

$$\sum_{a \in (A_m \cup A_{\bar{m}})} \vec{C}_a\left(\vec{F}_a^*\right)^T \cdot \left(\vec{F}_a - \vec{F}_a^*\right) - \sum_{w \in W} \vec{g}_w\left(\vec{T}_w^*\right)^T \cdot \left(\vec{T}_w - \vec{T}_w^*\right) \geq 0 \quad \forall \vec{F}, \vec{T} \in \Omega \tag{45}$$

where:

$$\vec{C}_a\left(\vec{F}_a\right)^T = \left[\ldots, \overline{CM}_{akl}\left(\vec{F}_a\right) + \theta\, t_{akl}\left(\vec{F}_a\right), \ldots\right]^T \tag{46}$$

$$\vec{g}_w\left(\vec{T}_w\right)^T = \left[-\frac{1}{\beta}\left(\ln T_w - qP_i\right) + \ln T_w\ ; \ldots, -\frac{1}{\beta_k}\left(\ln T_{wk} - \alpha_k\right) + \frac{1}{\gamma_k}\ln T_{wk}, \ldots ;\right.$$
$$\left.\ldots, -\frac{1}{\gamma_k}\ln T_{wkl} - \alpha_{kl} - R_{wkl} - \gamma_l\, l_{wkl} - \gamma_\sigma\, \sigma_{wkl}, \ldots\right]^T \tag{47}$$

Ω:

$$T_w = T_{wm} + T_{w\bar{m}} \quad \forall w \in W \tag{48}$$

$$T_{wm} = \sum_r T_{wmr} \quad \forall w \in W \tag{49}$$

$$T_{w\bar{m}} = \sum_d T_{w\bar{m}d} \quad \forall w \in W \tag{50}$$

$$T_{wmr} = \sum_{p \in P_{wmr}} h_p \quad \forall w \in W, \forall r \in R_m \qquad \left(U_{wmr}\right) \tag{51}$$

$$T_{w\bar{m}d} = \sum_{p \in P_{w\bar{m}d}} h_p \quad \forall w \in W, \forall d \in D_{\bar{m}} \qquad \left(U_{w\bar{m}d}\right) \tag{52}$$

$$O_i = \sum_j T_w \quad \forall i \in N_m \qquad \left(\lambda_i\right) \tag{53}$$

$$D_j = \sum_i T_w \quad \forall j \in \left\{N_m \cup N_{\bar{m}}\right\} \qquad \left(\lambda_j\right) \tag{54}$$

$$f_{amr} = \sum_w \sum_{p \in P_{wmr}} \delta_{ap} h_p \quad \forall a \in A_m, \forall r \in R_m \tag{55}$$

$$f_{a\bar{m}d} = \sum_w \sum_{p \in P_{w\bar{m}d}} \delta_{ap} h_p \quad \forall a \in \left\{A_m \cup A_{\bar{m}}\right\}, \forall d \in D_{\bar{m}} \tag{56}$$

$$\delta_{ap} = \begin{cases} 1 \text{ if } a \in p \\ 0 \text{ if } a \notin p \end{cases} \quad \forall a \in \left\{A_m \cup A_{\bar{m}}\right\}, \forall p \in P_w, \forall w \in W \tag{57}$$

$$T_w \geq 0, T_{wm} \geq 0, T_{w\bar{m}} \geq 0, T_{wmr} \geq 0, T_{w\bar{m}d} \geq 0 \quad \forall w \in W, \forall r \in R_m, \forall d \in D_{\bar{m}} \tag{58}$$

7. EQUILIBRIUM CONDITIONS

In this section we derive, for Model 3, the equilibrium conditions (3),(7),(8),(10),(18) and (19) by applying the necessary conditions for optimality, to the equivalent Lagrangean corresponding to the diagonalized formulation of problem P3. A similar demonstration is easier to do for Model 2 and is well known for Model 1.

7.1 Equivalent Lagrangean

We apply the necessary conditions for optimality to the Lagrangean function obtained by adjoining equations (51), (52), (53) and (54) to a diagonalized version of objective function (45). Equations (55) and (56) are used indirectly to apply the chain derivation rule and equations (48), (49) and (50) are used as definitions, in the derivation of the equilibrium conditions.

In the following expresion, $\bar{t}$ represents the diagonalized average travel time, perceived by each carrier, given the other flows corresponding to other carriers using the same infraestructure.

$$\begin{aligned}
\min_{(h,T)} L = & \sum_{a \in A_m} \sum_{r \in R_m} c_{amr}\, f_{amr} + \sum_{a \in \left(A_m \cup A_{\bar{m}}\right)} \sum_{d \in D_{\bar{m}}} c_{a\bar{m}d}\, f_{a\bar{m}d} \\
& + \phi \sum_{a \in A_m} \sum_{r \in R_m} \bar{t}_{amr}\left(f_{amr}\right) f_{amr} + \phi \sum_{a \in \left(A_m \cup A_{\bar{m}}\right)} \sum_{d \in D_{\bar{m}}} \bar{t}_{a\bar{m}d}\left(f_{a\bar{m}d}\right) f_{a\bar{m}d} \\
& + \theta \sum_{a \in A_m} \sum_{r \in R_m} \int_0^{f_{amr}} \bar{t}_{amr}(x)\, dx + \theta \sum_{a \in \left(A_m \cup A_{\bar{m}}\right)} \sum_{d \in D_{\bar{m}}} \int_0^{f_{a\bar{m}d}} \bar{t}_{a\bar{m}d}(x)\, dx \\
& + \frac{1}{\beta_m} \sum_w T_{wm}\left(\ln T_{wm} - 1 - \alpha_m\right) - \frac{1}{\gamma_m} \sum_w T_{wm}\left(\ln T_{wm} - 1\right) + \frac{1}{\gamma_m} \sum_w \sum_r T_{wmr}\left(\ln T_{wmr} - 1\right) \\
& + \frac{1}{\beta_{\bar{m}}} \sum_w T_{w\bar{m}}\left(\ln T_{w\bar{m}} - 1 - \alpha_{\bar{m}}\right) - \frac{1}{\gamma_{\bar{m}}} \sum_w T_{w\bar{m}}\left(\ln T_{w\bar{m}} - 1\right) + \frac{1}{\gamma_{\bar{m}}} \sum_w \sum_d T_{w\bar{m}d}\left(\ln T_{w\bar{m}d} - 1\right) \\
& + \sum_w \sum_r T_{wmr}\left(\alpha_{mr} + \varepsilon_{wmr} + \gamma_l\, l_{wmr} + \gamma_\sigma\, \sigma_{wmr}\right) + \sum_w \sum_d T_{w\bar{m}d}\left(\alpha_{\bar{m}d} + \varepsilon_{w\bar{m}d} + \gamma_l\, l_{w\bar{m}d} + \gamma_\sigma\, \sigma_{w\bar{m}d}\right) \\
& + \frac{1}{\beta} \sum_w T_w\left(\ln T_w - 1 - qP_i\right) - \sum_w T_w\left(\ln T_w - 1\right) \\
& + \sum_w U_{wmr}\left(T_{wmr} - \sum_{p \in P_{wmr}} h_p\right) + \sum_w U_{w\bar{m}d}\left(T_{w\bar{m}d} - \sum_{p \in P_{w\bar{m}d}} h_p\right) + \sum_i \lambda_i\left(O_i - \sum_j T_w\right) + \sum_j \lambda_j\left(D_j - \sum_i T_w\right)
\end{aligned} \tag{59}$$

7.2 Partial Derivatives

We first calculate the partial derivatives of the Lagrangean (59) with respect to the problem variables (flow variables: $h_p, \forall p \in P_w, \forall w \in W$, and demand variables: T_w, T_{wm}, $T_{w\bar{m}}$, T_{wmr}, $T_{w\bar{m}d}$, $\forall w \in W, \forall r \in R_m, \forall d \in D_{\bar{m}}$).

$$\frac{\partial L}{\partial h_p} = \overbrace{c_p + \phi\, t\overline{M}_p}^{\overline{CM}_p} + \theta\, \bar{t}_p - U_{wmr} \quad \forall p \in P_{wmr} \tag{60}$$

$$\frac{\partial L}{\partial h_p} = \overbrace{c_p + \phi\, t\overline{M}_p}^{\overline{CM}_p} + \theta\, \bar{t}_p - U_{w\bar{m}d} \quad \forall p \in P_{w\bar{m}d} \tag{61}$$

$$\frac{\partial L}{\partial T_w} = \frac{1}{\beta}\left(\ln T_w - qP_i\right) - \ln T_w - \lambda_i - \lambda_j \tag{62}$$

$$\frac{\partial L}{\partial T_{wm}} = \frac{1}{\beta_m}\left(\ln T_{wm} - \alpha_m\right) - \frac{1}{\gamma_m}\ln T_{wm} \tag{63}$$

$$\frac{\partial L}{\partial T_{w\bar{m}}} = \frac{1}{\beta_{\bar{m}}}\left(\ln T_{w\bar{m}} - \alpha_{\bar{m}}\right) - \frac{1}{\gamma_{\bar{m}}}\ln T_{w\bar{m}} \tag{64}$$

$$\frac{\partial L}{\partial T_{wmr}} = \frac{1}{\gamma_m}\ln T_{wmr} + \alpha_{mr} + \varepsilon_{wmr} + \gamma_l\, l_{wmr} + \gamma_\sigma\, \sigma_{wmr} + U_{wmr} \tag{65}$$

$$\frac{\partial L}{\partial T_{w\bar{m}d}} = \frac{1}{\gamma_{\bar{m}}}\ln T_{w\bar{m}d} + \alpha_{\bar{m}d} + \varepsilon_{w\bar{m}d} + \gamma_l\, l_{w\bar{m}d} + \gamma_\sigma\, \sigma_{w\bar{m}d} + U_{w\bar{m}d} \tag{66}$$

7.3 Equilibrium of Flows

Making derivatives (60) and (61), equal to zero, the network equilibrium conditions (18) and (19) are directly obtained:

$$\overline{CM}^*_p + \theta\, \bar{t}^*_p \begin{cases} = U^*_{wmr} & \text{if } h^*_p > 0 \\ \geq U^*_{wmr} & \text{if } h^*_p = 0 \end{cases} \quad \begin{matrix} w \in W \\ r \in R_m \\ p \in P_{wmr} \end{matrix} \tag{67}$$

$$\overline{CM}^*_p + \theta\, \bar{t}^*_p \begin{cases} = U^*_{w\bar{m}d} & \text{if } h^*_p > 0 \\ \geq U^*_{w\bar{m}d} & \text{if } h^*_p = 0 \end{cases} \quad \begin{matrix} w \in W \\ d \in D_{\bar{m}} \\ p \in P_{w\bar{m}d} \end{matrix} \tag{68}$$

7.4 Equilibrium of Trips by Carrier and Transfer Point

From derivatives (63) and (65) we obtain that:

$$\text{if } T^*_{wm} > 0 \Rightarrow \overbrace{\frac{1}{\beta_m}\left(\ln T^*_{wm} - \alpha_m\right)}^{X^*_{wm}} - \overbrace{\frac{1}{\gamma_m}\ln T^*_{wm}}^{Y^*_{wm}} = 0 \tag{69}$$

$$\text{if } T^*_{wmr} > 0 \Rightarrow \frac{1}{\gamma_m}\ln T^*_{wmr} + \alpha_{mr} + \varepsilon_{wmr} + \gamma_l l_{wmr} + \gamma_\sigma \sigma_{wmr} + U^*_{wmr} = 0 \tag{70}$$

Adding (69) and (70), isolating T^*_{wmr} and replacing the value of U^*_{wmr} from (67):

$$T^*_{wmr} = \exp -\gamma_m\left(X^*_{wm} - Y^*_{wm}\right)\exp -\gamma_m\left(\overline{CM}^*_p + \theta\,\bar{t}^*_p + \varepsilon_{wmr} + \gamma_l l_{wmr} + \gamma_\sigma \sigma_{wmr} + \alpha_{mr}\right) \tag{71}$$

Replacing the fare definition form R and using (9a) we get:

$$T^*_{wmr} = \exp - \gamma_m\left(X^*_{wm} - Y^*_{wm}\right)\exp - \underbrace{\gamma_m\left(R^*_{wmr} + \theta\bar{t}^*_p + \gamma_l l_{wmr} + \gamma_\sigma \sigma_{wmr} + \alpha_{mr}\right)}_{\gamma_m V^*_{wmr}} \tag{72}$$

Then dividing (72) by the sum, over al carrier r, of T^*_{wmr}, we can form: $\mathrm{G}^*_{\mathrm{wmr}} = \dfrac{T^*_{\mathrm{wmr}}}{\sum_{k\in R_m} {}^*_{\mathrm{wmk}}}$,

obtaining the equilibrium condition (7):

$$G^*_{wmr} = \frac{\exp\left(\gamma_m V^*_{wmr}\right)}{\sum_{k\in R_m}\exp\left(\gamma_m V^*_{wmk}\right)} \tag{73}$$

Making a similar development we obtain condition (8).

Equations (70) to (72) show that, for the derivation of consistent equilibrium conditions (see section 5) it is necessary that: i) fares charged by carriers be consistent with operating costs, ii) carrier network routing decisions, must consider shippers travel time cost. As we can see, this is the only way in which V^*_{wmr}, can be consistently replaced in (72).

Obviously if we change the definition of V^*_{wmr} in (9a), (9b), such that θt_p is not considered, then it is neither necessary to include such term in (67) and (68)[5]. Something similar happens with the transport fare R; it can only be replaced in (72) (to obtain V^*_{wmr}) if it is directly related to the operating cost c_p. The meaning is that, the fare charged to the shipper must ***at least*** recapture the cost incurred by the shipper in the provision of the transportation service. Then the only case in which the value of the fare could be arbitrary, is when the operating cost is zero.

7.5 Equilibrium of Trips by Mode

Replacing (72) in (49) we obtain:

$$T^*_{wm} = \sum_r\left[\exp -\gamma_m\left(X^*_{wm} - Y^*_{wm}\right)\exp\left(\gamma_m V^*_{wmr}\right)\right] \tag{74}$$

Then, simplifying, reordering and replacing the X^*_{wm} and Y^*_{wm} values we get:

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[5] Actually, if the term is included an inconsistency appears.

$$\frac{1}{\gamma_m}\ln\sum_r \exp\left(\gamma_m V^*_{wmr}\right) = \frac{1}{\beta_m}(\ln T^*_{wm} - \alpha_m) \tag{75}$$

Applying the definition of the generalized utility associated to each mode, and reordering terms, (75) can be written as:

$$\beta_m \tilde{V}^*_{wm} = \ln T^*_{wm} - \alpha_m \tag{76}$$

In a similar way we can obtain that:

$$\beta_{\bar{m}} \tilde{V}^*_{w\bar{m}} = \ln T^*_{w\bar{m}} - \alpha_{\bar{m}} \tag{77}$$

Subtracting (77) from (76) and using (48), we obtain equilibrium condition (10):

$$G^{-1*}_{wm} = -\ln\left(\frac{T^*_w}{T^*_{wm}} - 1\right) + \alpha_{\bar{m}} - \alpha_m \tag{78}$$

7.6 Equilibrium of Total Trips

From the expression of partial derivative (62), we obtain:

$$\text{if } T^*_w > 0 \Rightarrow \frac{1}{\beta}\left(\ln T^*_w - qP_i\right) - \ln T^*_w - \lambda^*_i - \lambda^*_j \tag{79}$$

Reordering and rising to the power *e* we get:

$$T^*_w = \exp\left(\beta\ \lambda^*_i\right)\exp\left(\beta\ \lambda^*_j\right)\exp\left(\beta\ \ln T^*_w + qP_i\right) \tag{80}$$

Then replacing (80) in (53) and (54) we obtain:

$$O_i = \exp\left(\beta\ \lambda^*_i\right)\sum_j \exp\left(\beta\ \lambda^*_j\right)\exp\left(\beta\ \ln T^*_w + qP_i\right) \tag{81}$$

$$D_j = \exp\left(\beta\ \lambda^*_j\right)\sum_i \exp\left(\beta\ \lambda^*_i\right)\exp\left(\beta\ \ln T^*_w + qP_i\right) \tag{82}$$

Now, let us define:

$$A^*_i = \frac{\exp\left(\beta\ \lambda^*_i\right)}{O_i} \quad \Rightarrow \quad \exp\left(\beta\ \lambda^*_i\right) = A^*_i O_i \tag{83}$$

$$B^*_i = \frac{\exp\left(\beta\ \lambda^*_j\right)}{D_j} \quad \Rightarrow \quad \exp\left(\beta\ \lambda^*_j\right) = B^*_j D_j \tag{84}$$

And taking the expressions of T^*_{wm} and $T^*_{w\bar{m}}$ from (76) and (77), we have:

$$T^*_{wm} = \exp\left(\beta_m \tilde{V}^*_{wm} + \alpha_m\right) \tag{85}$$

$$T^*_{w\bar{m}} = \exp\left(\beta_{\bar{m}} \tilde{V}^*_{w\bar{m}} + \alpha_{\bar{m}}\right) \tag{86}$$

Adding up both O-D demands and replacing in (48), then taking natural log, reordering and using the definition of generalized utility (logsum associated to each)-D pair, we get:

$$\ln T^*_w = \underbrace{\ln\left(\exp\left(\beta_m \tilde{V}^*_{wm} + \alpha_m\right) + \exp\left(\beta_{\bar{m}} \tilde{V}^*_{w\bar{m}} + \alpha_{\bar{m}}\right)\right)}_{L^*_w} \tag{87}$$

Finally, replacing (83), (84), and (87) in (80), we obtain the gravity model that describes the demand for O-D trips:

$$T_w^* = A_i^* O_i \; B_j^* D_j \; \exp\left(\beta \; L_w^* + qP_i \;\right) \tag{88}$$

7.7 Sufficient Conditions

The KUHN-TUCKER conditions (Zangwill, 1969), derived above are necessary and sufficient for the optimal solution of (P3) provided that the objective function of the diagonalized version of the objective function is convex, given that all the constraints are linear. By inspecting the expression of the Lagrangean (59), obtained from the diagonalized version of problem P3, it is easy to notice that only terms involving variables *T* may be nonconvex.

Therefore we must examine the second derivatives with respect to those variables. In particular, if we derive expression (62) with respect to T_w, we obtain:

$$\frac{1-\beta}{\beta} \cdot \frac{1}{T_w} \tag{89}$$

Now, deriving expressions (63) and (64) with respect to variables T_{wm} and $T_{w\bar{m}}$ we get:

$$\frac{\gamma_m - \beta_m}{\beta_m \gamma_m} \cdot \frac{1}{T_{wm}} \tag{90}$$

$$\frac{\gamma_{\bar{m}} - \beta_{\bar{m}}}{\beta_{\bar{m}} \gamma_{\bar{m}}} \cdot \frac{1}{T_{w\bar{m}}} \tag{91}$$

Finally, deriving (65) with respect to T_{wmr} and (66) with respect to $T_{w\bar{m}d}$ we obtain:

$$\frac{1}{\gamma_m} \frac{1}{T_{wmr}} \tag{92}$$

$$\frac{1}{\gamma_{\bar{m}}} \frac{1}{T_{w\bar{m}d}} \tag{93}$$

Therefore, from (89), (90), (91), (92) and (93) we have that given that all the variables *T* are always positive, P3 will be convex if: $\beta < 1$, $\beta_m < \gamma_m$, $\beta_{\bar{m}} < \gamma_{\bar{m}}$ and $\gamma_m > 0$, $\gamma_{\bar{m}} > 0$. Notice first that, according to the definition of the demand models, all parameters β and γ must be positive. In addition, the conditions requiring that $\beta < 1$ and $\beta < \gamma$ are consistent with the normal requirements for the validity of the hierarchical demand model assumed and represented in Figure 1 (see, Williams, 1977). Therefore, we will assume that such conditions are satisfied and therefore, the necessary conditions derived above (sections 7.3 to 7.6) for P3, are also sufficient.

8. SOLUTION APPROACH

Assuming that conditions (89), (90), (91) and (92) are satisfied, problem (P3) is a convex cost optimization problem. Therefore, similarly to many other network equilibrium models, it may be solved by the adaptation of the linear approximation algorithm of FRANK and WOLFE (Frank and Wolfe, 1956).

This approach is the most frequently used to solve transportation network problems and in particular has been successfully applied to obtain numerical solutions from performance-demand models similar to those proposed here (see Fernández et al. 1994).

The algorithm proposed has in general two main steps: i) the linear approximation subproblem which results from the linearization of the convex objective function and yields the descent direction and ii) the solution of a one-dimensional minimization problem, which determines the optimal step size to minimize the objective function, given the current solution and the descent direction.

An specially useful and intuitively appealing variant of the algorithm proposed is the so called partial linearization algorithm (see Evans, 1976), where only the terms involving the link flows are linearized and the terms related to demand variables are not. Given the relative large number of demand terms on the objective function of problem (P3), it can be specially advantageous to use also the Horowitz approximation (Horowitz, 1989).

The necessary steps to solve problem (P3) will be the following: The minimum cost paths (based on the current feasible solution) would be computed for each carrier *r* on each network, corresponding to pure and combined modes. For this, transfers, $d \in D_{\bar{m}}$, are considered as additional origin and destination points for the corresponding modes: then combined mode minimum cost paths are computed through each transfer point for each O-D pair. Then, using (9a) and (9b), generalized costs perceived by shippers are calculated and the corresponding EMUs, V_{wm} and $V_{w\bar{m}}$, and Logsums, L_w are also computed. Next, new demands *T* are obtained applying demand models (3), (7), (8), (10), (12a) and (12b). Finally, the resulting O-D pair trips would be assigned to each of the networks, according to the minimum cost paths calculated before. Once an auxiliary solution has been obtained by the procedure described, a convex combination of this with the current solution is computed. For this, a one-dimensional search must be performed in the direction defined by the current and auxiliary solutions; this task can be importantly simplified, if only the network related terms of the objective function are considered, according to the use of the Horowitz approximation.

We omit here the technical details of the adaptations proposed, since they are tedious and do not add much insight to the structure of problem (P3). It is clear however that by the application of the general procedure described it is possible to compute solutions on large scale networks.

9. CONCLUSIONS AND FURTHER RESEARCH

We have presented three different demand-supply equilibrium formulations to the modeling of interurban, multimodal, freight transportation systems. Each of these formulations is based on different assumptions, regarding the costs perceptions of shippers and carriers and represent different levels of approximation to the operation of real interurban freight transportation systems. The level of complexity of the analysis is correspondingly different in each case, going from the simplest case, without congestion in the first formulation, to the full complexity of the third model, in which most of the relevant factors present in the real decision process are considered.

It is specially interesting to notice that full consideration of congestion effects in the simultaneous equilibrium formulation, proposed in Model 3, is possible only under some important consistency requirements: i) Transportation fares paid by shippers must be related to the operating costs experienced by carriers; in the formulation proposed, fares are equal to marginal costs plus profit, ii) Routing decisions, taken by carriers, which determine the level of service offered by them, must take into account shippers preferences with respect to travel times. In other words, carriers must be rational in a microeconomic sense, if we are to use a simultaneous equilibrium formulation.

Model 3, corresponds to an entirely new, more consistent general network formulation for the simulation of interurban freight transportation systems; it allows to calibrate different parameters for the choices of mode, carrier and transfer point and considers that they are based on level of service values that are consistent with the simulation of network operations.

The mathematical formulations derived result in computable equilibrium flows in each case and in the last section a solution algorithm to obtain them has been outlined. It remains to develop and study alternative solution approaches for the formulations proposed. The analysis of such solution methods and their practical consequences for real applications will be the subject of forthcoming papers.

ACKNOWLEDGMENTS

This research was financed by a grant from the Chilean National Research Fund (FONDECYT) and by the Catholic University of Chile.

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